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How Can We Obtain A Large Majorana-Mass in Calabi-Yau Models ?

Naoyuki HABA,[1] Chuichiro HATTORI,[2] Masahisa MATSUDA,[3]
Takeo MATSUOKA[1] and Daizo MOCHINAGA[1]

[1] *Department of Physics, Nagoya University*
Nagoya, JAPAN 464-01

[2] *Department of Physics, Aichi Institute of Technology*
Toyota, Aichi, JAPAN 470-03

[3] *Department of Physics and Astronomy*
Aichi University of Education
Kariya, Aichi, JAPAN 448

Abstract

In a certain type of Calabi-Yau superstring models it is clarified that the symmetry breaking occurs by stages at two large intermediate energy scales and that two large intermediate scales induce large Majorana-masses of right-handed neutrinos. Peculiar structure of the effective nonrenormalizable interactions is crucial in the models. In this scheme Majorana-masses possibly amount to $O(10^{9\sim10}\text{GeV})$ and see-saw mechanism is at work for neutrinos. Based on this scheme we propose a viable model which explains the smallness of masses for three kind of neutrinos ν_e, ν_μ and ν_τ. Special forms of the nonrenormalizable interactions can be understood as a consequence of an appropriate discrete symmetry of the compactified manifold.

1 Introduction

While superstring theory is the only known candidate of consistent unification of all fundamental interactions, untill now we have not succeeded in selecting a true string vacuum theoretically. This is because we are lacking a means of addressing the non-perturbative problems. In such situation of superstring theory it is valuable to clarify how to connect superstring theory with the standard model and to understand phenomenological implications of the effective theory from superstring theory. As a matter of fact, by using phenomenological requirements on superstring-derived models we can classify the string vacua corresponding to a huge number of distinct classical solutions. It is expected that further study along this point of view provides an important clue to find a true string vacuum.

In Calabi-Yau superstring models, unlike the standard gauge group $G_{st} = SU(3)_c \times SU(2)_L \times U(1)_Y$ with rank-four, the gauge group is rank-six or rank-five at the compactification scale M_C[1]. In the followings we discuss rank-six models coming from abelian flux breaking. Consequently, there should exist two intermediate energy scales of symmetry breaking between the compactification scale and the electroweak scale. In Calabi-Yau models there appear extra matter fields which are not contained in the minimal supersymmetric standard model. We generally have G_{st}-neutral but E_6-charged chiral superfields and their mirror chiral superfields. Concretely we get $SO(10)$-singlet chiral superfields and $SU(5)$-singlet chiral superfields (right-handed neutrino ν_R^c) denoted as S and N, respectively, which belong to **27**-representation of E_6. Some of these G_{st}-neutral matter fields have to develop non-vanishing vacuum expectation values(VEVs) $\langle S \rangle$ and $\langle N \rangle$ at the intermediate energy scales in order to connect the Calabi-Yau models with the standard model.

To solve the so-called hierarchy problem, it is natural that the supersymmetry(susy) is preserved down to an energy scale as low as $O(10^3\text{GeV})$. From phenomenological point of view it is well known that there are at least two large energy scales between the Planck scale and the soft susy breaking scale $m_{SUSY} = O(10^3\text{GeV})$. These scales are concerned with the proton decay and a large Majorana-mass(M-

mass) of the right-handed neutrino.

As for the former subject, in Calabi-Yau models the lifetime of proton is determined by the magnitude of $\langle S \rangle$, because the superfield S participates in a Yukawa interaction with leptoquark chiral superfields. To be consistent with the proton stability, it is normally required that $\langle S \rangle \geq O(10^{16}\mathrm{GeV})$. Although this condition can be somewhat relieved provided that the sparticle spectrum is tuned adequately, even in the case $\langle S \rangle \geq O(10^{14}\mathrm{GeV})$ is required [2].

The latter subject is related to see-saw mechanism. Experimentally neutrino masses are so small compared with quark masses and charged lepton masses [3]. See-saw mechanism provides an interesting solution for the neutrino mass problem by introducing large M-masses for right-handed neutrinos. If we take the solar neutrino problem seriously, the M-mass of the right-handed neutrino should be of order $10^{9\sim 12}$GeV[4] [5]. Also this large M-mass is compatible with the cosmological bound for stable light neutrinos [6]. Since a non-vanishing $\langle N \rangle$ implies the lepton number violation, the magnitude of $\langle N \rangle$ seems to be closely linked to a M-mass of the right-handed neutrino. A large M-mass suggests a large value of $\langle N \rangle$.

When $\langle S \rangle, \langle N \rangle \gg m_{SUSY}$, we have to make the D-terms vanish at such large scales $\langle S \rangle$ and $\langle N \rangle$. This is realized by setting $\langle S \rangle = \langle \overline{S} \rangle$ and $\langle N \rangle = \langle \overline{N} \rangle$, where $\overline{S}$ and $\overline{N}$ stand for mirror chiral superfields of S and N, respectively. How can we derive such large intermediate scales in Calabi-Yau superstring models? The discrete symmetry of the compactified manifold possibly accomplishes this desired situation [7]. In superstring models there exist effective non-renormalizable(NR) terms in the superpotential. The order of magnitudes of $\langle S \rangle$ and $\langle N \rangle$ are governed by these NR terms. Along this fascinating line the problems of two large intermediate scales of symmetry breaking have been studied first by Masip [8]. In the analysis general structure of the scalarpotential has not been sufficiently clarified. So conditions on the NR terms for the presence of two large intermediate scales and of a large M-mass should be studied.

In this paper, we study the NR terms in the superpotential which satisfy the

following two requirements. The first one is the presence of two large intermediate energy scales of symmetry breaking. The second one is the presence of a large M-mass of $O(10^{9\sim12}\mathrm{GeV})$. The solutions which meet these requirements are found only in the case when the NR terms are of special forms. Concretely, the NR interactions of S, N and $\overline{S}, \overline{N}$ are of the form

$$W_{NR} = {M_C}^3 \lambda_1 \left[\left(\frac{S\overline{S}}{{M_C}^2} \right)^{2k} + k \left(\frac{N\overline{N}}{b^2\, {M_C}^2} \right)^2 - 2c \left(\frac{S\overline{S}}{{M_C}^2} \right)^k \left(\frac{N\overline{N}}{b^2\, {M_C}^2} \right) \right] \tag{1}$$

with $k = 3, 4, \ldots$ and $0 < c < \sqrt{2k}$ and $c \neq \sqrt{k}$, where λ_1 and b are real constants of $O(1)$. As a result we have two large intermediate scales

$$\langle S \rangle \geq O(10^{16}\mathrm{GeV}), \qquad O(10^{15}\mathrm{GeV}) \geq \langle N \rangle \geq O(10^{13}\mathrm{GeV}) \tag{2}$$

and a M-mass of right-handed neutrino becomes

$$M_M \sim m_{SUSY} \left(\frac{\langle S \rangle}{\langle N \rangle} \right)^2 . \tag{3}$$

Its numerical value possibly amounts to $O(10^{9\sim10}\mathrm{GeV})$. Thus see-saw mechanism is at work and this large M-mass solves the solar neutrino problem. The main results have been presented in the previous paper by the present authors [12].

This paper is organized as follows. In Sec. 2 we discuss the connection between the NR interactions and intermediate scales of symmetry breaking. In the presence of the NR interactions we get a M-mass matrix by means of minimization conditions of the scalarpotential. We require solutions to imply the existence of two large intermediate scales and of a large M-mass. In Sec. 3 we look for solutions which meet the requirements. As a consequence, special types of the NR terms are selected. The solutions obtained there correspond to a local minimum of the scalarpotential but not necessarily to the absolute minimum. The structure of the scalarpotential is studied in detail for the special types of the NR interactions in Sec. 4. Under an adequate condition it is shown that the desirable solution represents the absolute minimum of the scalarpotential. M-masses are obtained concretely. To get a M-mass with $O(10^{9\sim10}\mathrm{GeV})$, the form of the NR terms are further sorted. Taking the generation

degree of freedom into account, in Sec. 5 we propose a viable model which explains the smallness of masses for three kind of neutrinos ν_e, ν_μ and ν_τ. The final section is devoted to summary and discussion.

2 Intermediate Scales of Symmetry Breaking

Before examining in the scheme that $S, \overline{S}$ and $N, \overline{N}$ appear in the massless spectra at the compactification scale M_C, for illustration we first study the NR interactions coming from only a pair of S and $\overline{S}$ chiral superfields. The NR terms in the superpotential are of the form

$$W_{NR} = \sum_{p=2}^{\infty} \lambda_p {M_C}^{3-2p} (S\overline{S})^p, \tag{4}$$

where dimensionless coupling λ_p's are of order one. However, if the compactified manifold has a specific type of discrete symmetry, some of λ_p's become vanishing. When we denote the lowest number of p as n, the NR terms are approximately written as

$$W_{NR} \cong \lambda_n {M_C}^{3-2n} (S\overline{S})^n, \tag{5}$$

because the terms with larger p are suppressed by the inverse power of M_C at low energies. In the three-generation model obtained from the Tian-Yau manifold or the Schimmrigk manifold we have $n = 2,\ 3$ [9] [10] [11]. While in the four-generation model with the high discrete symmetry $S_5 \times {Z_5}^5$, this symmetry leads to $n = 4$ [7].

To maintain susy down to a TeV scale, the scalarpotential should satisfy F-flatness and D-flatness conditions at the large intermediate scale. Then we have to set $\langle S\rangle = \langle\overline{S}\rangle$. As far as D-terms are concerned, the VEV can be taken as large as we want. Incorporating the soft susy breaking terms, we have the scalarpotential

$$\begin{aligned} V &= n^2 {\lambda_n}^2 {M_C}^{6-4n} \left(|S|^{2(n-1)} |\overline{S}|^{2n} + |S|^{2n} |\overline{S}|^{2(n-1)} \right) \\ &\qquad + \frac{1}{2} \sum_\alpha {g_\alpha}^2 \left(S^\dagger T_\alpha S - \overline{S}^\dagger T_\alpha \overline{S} \right)^2 + V_{soft}, \end{aligned} \tag{6}$$

$$V_{soft} = {m_S}^2 \, |S|^2 + m_{\overline{S}}^2 \, |\overline{S}|^2, \tag{7}$$

where the T_α are Lie algebra generators and ${m_S}^2$ and ${m_{\overline{S}}}^2$ are the running scalar masses squared associated with the soft susy breaking. S and $\overline{S}$ develop nonzero VEVs when ${m_S}^2 + {m_{\overline{S}}}^2 < 0$. In the renormalization group analysis it has been proven that ${m_S}^2 + {m_{\overline{S}}}^2$ possibly becomes negative at the large intermediate scale $O(10^{16}\mathrm{GeV})$ [13]. By minimizing V, we obtain the VEVs as

$$\langle S \rangle \simeq \langle \overline{S} \rangle \sim M_C \left(\frac{\sqrt{-m_S^2}}{M_C} \right)^{1/2(n-1)} . \tag{8}$$

The difference $\langle S \rangle - \langle \overline{S} \rangle$ is negligibly small and we put ${m_S}^2 = {m_{\overline{S}}}^2$ approximately. In the case $n = 3,\ 4$ the intermediate energy scale becomes $\langle S \rangle \simeq \langle \overline{S} \rangle \sim O(10^{14}\mathrm{GeV})$, $O(10^{16}\mathrm{GeV})$, respectively, for $M_C = 10^{18\sim19}\mathrm{GeV}$. If $n = 2$, then we have $\langle S \rangle \sim 10^{11}\mathrm{GeV}$, which leads to the fast proton decay. Through the super-Higgs mechanism, the $(S - \overline{S})/\sqrt{2}$ are absorbed into a massive vector superfield with its mass of $O(g_\alpha \langle S \rangle)$. The component $(S+\overline{S})/\sqrt{2}$ have masses of order $O(10^3\mathrm{GeV})$ irrespectively of n. In addition to $\langle S \rangle$ and $\langle \overline{S} \rangle$, we need $\langle N \rangle$ and $\langle \overline{N} \rangle$ in order to get sufficiently large M-masses relative to the soft susy breaking scale.

Next we turn to investigate the case in which the NR terms consist of pairs of S, N and $\overline{S}$, $\overline{N}$ chiral superfields, provided that there appear S, N and $\overline{S}$, $\overline{N}$ superfields in suitable Calabi-Yau models. Here we assume the NR interactions

$$W_{NR} = {M_C}^3 \left[\lambda_1 \frac{(S\overline{S})^n}{{M_C}^{2n}} + \lambda_2 \frac{(N\overline{N})^m}{{M_C}^{2m}} + \lambda_3 \frac{(S\overline{S})^k (N\overline{N})^l}{{M_C}^{2(k+l)}} \right], \tag{9}$$

where n, m, k and l are integers with

$$n > k \geq 1, \qquad m > l \geq 1 \tag{10}$$

and λ_i's are real constants of $O(1)$. In certain types of Calabi-Yau models it is plausible that the exponents n, m, k and l are settled on appropriate values due to the discrete symmetry of the Calabi-Yau manifold. In this scheme we potentially derive two intermediate energy scales of symmetry breaking and possibly have a large M-mass. By minimizing the scalarpotential including the soft susy breaking terms

$$V_{soft} = {m_S}^2\,|S|^2 + {m_{\overline{S}}}^2\,|\overline{S}|^2 + {m_N}^2\,|N|^2 + {m_{\overline{N}}}^2\,|\overline{N}|^2, \tag{11}$$

we can determine the energy scales of symmetry breaking, that is, $\langle S\rangle$ and $\langle N\rangle$. The scalar mass parameters ${m_S}^2$ and ${m_N}^2$ evolve according to the renormalization group equations. As shown in ref.[13], we expect that ${m_S}^2$ becomes negative at the large intermediate scale(M_I). On the other hand, it is natural to expect that ${m_N}^2$ remains still positive at M_I scale, because quantum numbers and Yukawa interactions of N and $\overline{N}$ are quite different from those of S and $\overline{S}$. For this reason we consider the case $|\langle S\rangle| \gg |\langle N\rangle|$. Hereafter we take ${m_S}^2({m_{\overline{S}}}^2) < 0$ and ${m_N}^2({m_{\overline{N}}}^2) > 0$ at M_I scale. However, the sign of ${m_N}^2$ is not crucial in the following discussions. From the D-flatness condition we get $|\langle S\rangle| = |\langle \overline{S}\rangle|$ and $|\langle N\rangle| = |\langle \overline{N}\rangle|$ in the approximation ${m_S}^2 = {m_{\overline{S}}}^2$ and ${m_N}^2 = {m_{\overline{N}}}^2$. Here we assume that the VEVs are expressed as

$$\langle S\rangle = \langle \overline{S}\rangle = M_C\, x, \qquad \langle N\rangle = \langle \overline{N}\rangle = M_C\, y. \tag{12}$$

Without loss of generality x is taken as real for simplicity. For convenience' sake, instead of λ_i's we use the parameters a, b and c defined as

$$\lambda_1 = \frac{a}{n}, \qquad \lambda_2 = \frac{a}{m}\, b^{-2m}, \qquad \lambda_3 = -\frac{ac}{kl}\, b^{-2l}, \tag{13}$$

where a is real. When $\lambda_2/\lambda_1 > 0$, b and c can be put as real. For positive c there possibly exist solutions with real y as seen later. In the case with negative c, if and only if m and l are even and odd, respectively, we can reduce this case to the case with positive c by redefining the fields N and $\overline{N}$ attached by a phase factor i as N and $\overline{N}$. When $\lambda_2/\lambda_1 < 0$, b becomes complex. However, if we redefine the fields N and $\overline{N}$ multiplied by an adequate phase factor as N and $\overline{N}$ and then if c becomes real, this case can be again reduced to the above-mentioned case. Otherwise, we do not have desirable solutions. In what follows we put b and c as real and positive and then y is taken as real. Let us introduce dimensionless real functions f and g :

$$\begin{aligned} f(x,y) &\equiv {M_C}^{-2} \left.\frac{\partial W}{\partial S}\right| = X + Z_f, \\ g(x,y) &\equiv {M_C}^{-2} \left.\frac{\partial W}{\partial N}\right| = Y + Z_g \end{aligned} \tag{14}$$

with

$$X = a x^{2n-1}, \qquad Z_f = -\frac{ac}{l} x^{2k-1} \left(\frac{y}{b}\right)^{2l},$$
$$Y = \frac{a}{b} \left(\frac{y}{b}\right)^{2m-1}, \qquad Z_g = -\frac{ac}{kb} x^{2k} \left(\frac{y}{b}\right)^{2l-1}, \tag{15}$$

where $\ldots|$ means the values at $S = \overline{S} = \langle S \rangle$ and $N = \overline{N} = \langle N \rangle$. By using the D-flatness condition we have the scalarpotential

$$\frac{1}{2} {M_C}^{-4} V| = f(x,y)^2 + g(x,y)^2 - {\rho_x}^2 x^2 + {\rho_y}^2 y^2 \tag{16}$$

with

$${\rho_x}^2 = -\frac{{m_S}^2}{{M_C}^2} \ (> 0), \qquad {\rho_y}^2 = \frac{{m_N}^2}{{M_C}^2} \ (> 0). \tag{17}$$

Since ρ_x and ρ_y are of order $O(10^{-(15\sim16)})$, hereafter we often denote ρ_x and ρ_y simply as a positive parameter $\rho (= O(m_{SUSY}/M_C))$ together.

We are going to carry out the minimization of the scalarpotential V. Since the scalarpotential is symmetric under the reflection $x \to -x$ and/or $y \to -y$, it is sufficient for us to consider only the first quadrant in the x-y plane. The solution of interest here is the one which implies two large intermediate scales of symmetry breaking with $\langle S \rangle \gg \langle N \rangle \gg m_{SUSY}$. At the absolute minimum stationary conditions

$$\left.\frac{\partial V}{\partial S}\right| = \left.\frac{\partial V}{\partial \overline{S}}\right| = \left.\frac{\partial V}{\partial N}\right| = \left.\frac{\partial V}{\partial \overline{N}}\right| = 0 \tag{18}$$

have to be satisfied. These conditions are expressed as

$$f\, f_x + g\, g_x - {\rho_x}^2\, x = 0,$$
$$f\, f_y + g\, g_y + {\rho_y}^2\, y = 0, \tag{19}$$

where $f_x = \partial f / \partial x$ and so forth. More explicitly, we have

$$f_x = \frac{1}{x}[(2n-1)X + (2k-1)Z_f],$$
$$f_y = g_x = \frac{2l}{y} Z_f = \frac{2k}{x} Z_g, \tag{20}$$
$$g_y = \frac{1}{y}[(2m-1)Y + (2l-1)Z_g].$$

For $S, \overline{S}$ and $N, \overline{N}$ the mass matrix is given by

$$\begin{pmatrix} W_{SS}| & W_{S\overline{S}}| & W_{SN}| & W_{S\overline{N}}| \\ W_{\overline{S}S}| & W_{\overline{S}\overline{S}}| & W_{\overline{S}N}| & W_{\overline{S}\overline{N}}| \\ W_{NS}| & W_{N\overline{S}}| & W_{NN}| & W_{N\overline{N}}| \\ W_{\overline{N}S}| & W_{\overline{N}\overline{S}}| & W_{\overline{N}N}| & W_{\overline{N}\overline{N}}| \end{pmatrix}, \tag{21}$$

where $W_{SS} = \partial^2 W/\partial S^2$ and so forth. Through the super-Higgs mechanism the components $(S - \overline{S})/\sqrt{2}$ and $(N - \overline{N})/\sqrt{2}$ are absorbed by vector superfields which then become massive with masses $g_\alpha \langle S \rangle$ and $g_\alpha \langle N \rangle$, respectively. The remaining components $(S + \overline{S})/\sqrt{2}$ and $(N + \overline{N})/\sqrt{2}$ become Majorana superfields. The mass matrix for $(S + \overline{S})/\sqrt{2}$ and $(N + \overline{N})/\sqrt{2}$ denoted as $M_C\, A$ is of the form

$$\begin{aligned} M_C\, A &\equiv \begin{pmatrix} W_{SS}| + W_{S\overline{S}}| & 2W_{SN}| \\ 2W_{SN}| & W_{NN}| + W_{N\overline{N}}| \end{pmatrix} \\ &= M_C \begin{pmatrix} f_x & g_x \\ f_y & g_y \end{pmatrix} \end{aligned} \tag{22}$$

with $g_x = f_y$. Here we used the relations

$$\begin{aligned} &W_{SS}| = W_{\overline{S}\overline{S}}|, \qquad W_{NN}| = W_{\overline{N}\overline{N}}|, \\ &W_{SN}| = W_{S\overline{N}}| = W_{\overline{S}N}| = W_{\overline{S}\overline{N}}|. \end{aligned} \tag{23}$$

Since the matrix A is real and symmetric, we can diagonalize this matrix via an orthogonal transformation. By using the matrix A, we can rewrite the stationary conditions Eq. (19) in the matrix form

$$A \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} {\rho_x}^2 x \\ -{\rho_y}^2 y \end{pmatrix}. \tag{24}$$

In the next section we solve Eq. (24) in order to find the absolute minimum. We look for the solution in which $x \gg y \neq 0$ and also a M-mass becomes sufficiently large relative to $O(10^3\text{GeV})$. We obtain the constraint on the NR terms for the existence of desirable solutions. The constraint yields a relation among the exponents n, m, k and l.

3 Solutions with A Large Majorana-Mass

We are now going to find a solution which corresponds to the absolute minimum with $x \gg y \neq 0$. Since the value of the scalarpotential should be negative at the point, we have

$$f^2 + g^2 < {\rho_x}^2 x^2 - {\rho_y}^2 y^2 \sim \rho^2\, x^2 \tag{25}$$

for the solution, where we used the relation $x^2 \gg y^2$. Furthermore, it can be shown that

$$f^2 + g^2 = O(\rho^2 x^2). \tag{26}$$

If it were not for the case, we have $f^2 + g^2 \ll \rho^2\, x^2$. This implies that $|f|,\ |g| \ll \rho\, x$. On the other hand, from Eq. (19) we get $f\, x f_x + g\, x g_x \sim \rho^2 x^2$. Then it is impossible that both $|f\, x f_x|$ and $|g\, x g_x|$ are smaller than $O(\rho^2 x^2)$.

If $|f\, x f_x| \gtrsim \rho^2 x^2$, we have $|x f_x| \gg \rho x \gg |f|$. This means that the cancellation of the leading terms of X and Z_f occurs in f. In this case we get $|Z_f| \sim |x f_x| \gg \rho\, x$. Thus $|Z_g| \sim (x/y)|Z_f| \gg \rho x^2/y \gg \rho x \gg |g|$. This means that the cancellation occurs also between Y and Z_g in g. However, the cancellation of the leading terms both in f and g results in a high degree of fine tuning which we consider unlikely. In fact, by eliminating x in the relations $f = X + Z_f \sim 0$ and $g = Y + Z_g \sim 0$ we obtain

$$\left(\frac{y}{b}\right)^{2(nm-nl-mk)} = \left(\frac{k}{l}\right)^k \left(\frac{k}{c}\right)^n \tag{27}$$

at the leading order. In the case the exponent $mn - nl - mk \neq 0$, x and y turn out to be expressed as functions only of b and c. By substituting these into Eq. (19) we have relations between b, c and ρ_x, ρ_y. Parameters ρ_x and ρ_y are the running ones of the soft susy breaking determined by the renormalization group equations. While b and c are coupling constants of the NR terms in superpotential. Therefore, these relations imply a fine tuning which we consider unlikely. In the case $mn - nl - mk = 0$, c is fixed to a specific value. However, it is also unlikely that such a special value of c is derived from the discrete symmetry of the compactified manifold.

Next we consider the case $|f\,xf_x| \ll \rho^2 x^2$ and then $|g\,xg_x| \sim \rho^2 x^2$. Similarly to the above argument, we get $|xg_x| \gg \rho x \gg |g|$. Then the cancellation of the leading terms of Y and Z_g have to take place in g. Since this means $|yg_y| \sim |xg_x|$, we obtain $|g\,yg_y| \sim |g\,xg_x| \sim \rho^2 x^2$. While, from Eq. (19) we have $f\,yf_y + g\,yg_y \sim \rho^2 y^2$. In order to satisfy this relation under $x^2 \gg y^2$, $|f\,yf_y| \sim |g\,yg_y| \sim \rho^2 x^2$ and the leading terms of $f\,yf_y$ and $g\,yg_y$ have to cancel out with each other. In this case we get $|yf_y| \gg \rho x \gg |f|$ and then $|Z_f| \sim |xf_x| \sim |yf_y|$. Thus $|f\,xf_x| \sim \rho^2 x^2$. This contradicts with the relation supposed here. Therefore, we obtain the relation (26).

Next we show that $|f| = O(\rho x)$ and that only one M-mass possibly becomes large compared with m_{SUSY}. Through an orthogonal transformation we carry out the diagonalization of the matrix A as

$$U\,A\,U^{-1} = \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix}, \tag{28}$$

where

$$U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{29}$$

Then M-masses are $M_C|\omega_1|$ and $M_C|\omega_2|$. We require that at least one of $|\omega_1|$ and $|\omega_2|$ is sufficiently larger than $O(\rho) = O(m_{SUSY}/M_C)$. However, it is impossible that both $|\omega_1|$ and $|\omega_2|$ are larger than $O(\rho)$. To see this, let us suppose for a moment that both $|\omega_1|$ and $|\omega_2|$ are larger than $O(\rho)$, i.e., $O(\rho) \ll |\omega_1| \leq |\omega_2|$. From Eq. (24) we have

$$\begin{aligned}
\left({\rho_x}^2\,x\right)^2 + \left({\rho_y}^2\,y\right)^2 &= \begin{pmatrix} f & g \end{pmatrix} A^T A \begin{pmatrix} f \\ g \end{pmatrix} \\
&= {\omega_1}^2\,\left(f\cos\theta - g\sin\theta\right)^2 + {\omega_2}^2\,\left(f\sin\theta + g\cos\theta\right)^2 \\
&\geq {\omega_1}^2 \left(f^2 + g^2\right). \\
&\sim {\omega_1}^2 \rho^2 x^2,
\end{aligned}$$

where we used Eq. (26). This is inconsistent with the relation supposed here. Thus we have to be

$$|\omega_1| \leq O(\rho), \qquad |\omega_2| \gg O(\rho). \tag{30}$$

From Eq. (28) A is expressed as

$$A = \begin{pmatrix} f_x & g_x \\ f_y & g_y \end{pmatrix} = \begin{pmatrix} \omega_1 \cos^2\theta + \omega_2 \sin^2\theta & (\omega_2 - \omega_1)\sin\theta\cos\theta \\ (\omega_2 - \omega_1)\sin\theta\cos\theta & \omega_1 \sin^2\theta + \omega_2 \cos^2\theta \end{pmatrix}. \tag{31}$$

Then we obtain

$$f_x = \omega_1 \cos^2\theta + \omega_2 \sin^2\theta, \tag{32}$$

$$f_y = (\omega_2 - \omega_1)\sin\theta\cos\theta. \tag{33}$$

Unless $|\sin\theta| \ll 1$, it turns out that $|\omega_2|\sin^2\theta \gg |\omega_1|\cos^2\theta$ because of Eq. (30). By using $Z_f \sim y f_y$ and Eq. (33), we get $|x f_x| \sim |\omega_2| x \sin^2\theta \gg |\omega_2 y \sin\theta\cos\theta| \sim |Z_f|$. This implies that $|X| \gg |Z_f|$ and then $|f| \sim |x f_x| \sim |\omega_2| x \sin^2\theta \gg \rho x$. This contradicts with Eq. (26). Thus we are led to the inequality

$$|\sin\theta| \ll 1. \tag{34}$$

Without loss of generality we can take $|\theta| \ll 1$. Then the matrix A is approximated as

$$A = \begin{pmatrix} f_x & g_x \\ f_y & g_y \end{pmatrix} \simeq \begin{pmatrix} \omega_1 + \omega_2\theta^2 & \omega_2\theta \\ \omega_2\theta & \omega_2 \end{pmatrix}. \tag{35}$$

Combining Eq. (24) with this expression we obtain

$$(\omega_1 + \omega_2\theta^2) f + \omega_2\theta g \simeq {\rho_x}^2 x, \tag{36}$$

$$\omega_2\theta f + \omega_2 g \simeq -{\rho_y}^2 y. \tag{37}$$

Subtracting Eq. (37) multiplied by θ from Eq. (36), we find

$$\omega_1 f \simeq {\rho_x}^2 x. \tag{38}$$

In consideration of Eqs. (26) and (30), this leads us to

$$|\omega_1| = O(\rho) \tag{39}$$

and $|f| \sim \rho x$. Therefore, Eqs. (36) and (37) are translated as

$$|f| \sim \rho x, \tag{40}$$

$$\theta f + g \sim -\frac{\rho^2}{\omega_2} y \tag{41}$$

with $|\omega_2| \gg O(\rho)$.

From Eq. (35) we get

$$f_x \sim \rho + \omega_2 \theta^2, \tag{42}$$

$$f_y = g_x \sim \omega_2 \theta, \tag{43}$$

$$g_y \sim \omega_2. \tag{44}$$

To solve these equations together with Eqs. (40) and (41), it is convenient for us to classify into two cases according to whether or not the cancellation between the leading terms of Y and Z_g occurs in g. First consider the case when there is no cancellation in g. Taking Eqs. (43) and (44) into account, we can compare the magnitude of each term in Eq.(41). On the left hand side of Eq. (41) $|g|$ is sufficiently larger than $|\theta f|$, because $|g| \gtrsim |Z_g| \sim |x g_x| \sim |\theta\omega_2| x \gg |\theta|\rho x \sim |\theta f|$. While the right hand side of Eq. (41) is much smaller than $|g|$, i.e., $|\rho^2 y/\omega_2| \ll \rho y \ll |\omega_2| y \sim |y g_y| \lesssim |g|$, where we used Eq. (44). Then Eq. (41) can not be satisfied in this case. Therefore, a cancellation of the leading terms in $g = Y + Z_g$ have to take place and a cancellation does not occur in g_y. Thus

$$|x g_x| \sim |Z_g| \sim |Y| \sim |y g_y|. \tag{45}$$

Using Eqs. (43) and (44), we get $|\omega_2\theta| x \sim |\omega_2| y$. This means that

$$|\theta| \sim \frac{y}{x}. \tag{46}$$

From Eq. (31) we have $g_y = \omega_2 + O(\omega_2\theta^2)$. Since the next-to-leading term is suppressed by θ^2 relative to the leading one, we can express as

$$|g| \sim \theta^2 |Y|. \tag{47}$$

The magnitude of each term in Eq. (41) is estimated as

$$|\theta f| \sim |\theta| \rho x \sim \rho y, \tag{48}$$

$$|g| \sim |\theta^2 Y| \sim |\theta^2 y g_y| \sim |\theta^2 \omega_2| y, \tag{49}$$

$$\left| -\frac{\rho^2}{\omega_2} y \right| \ll \rho y. \tag{50}$$

Consequently, in order that Eq. (41) holds, the leading terms of θf and g have to cancel out with each other. Thus from Eqs. (48) and (49) we obtain

$$\theta^2 \sim \frac{\rho}{|\omega_2|}. \tag{51}$$

Returning to Eqs. (42) and (43), we get

$$|f| \sim |X| \sim |Z_f|. \tag{52}$$

The conclusion of this section is that a desirable solution exists only in the case when a cancellation of the leading terms occurs in g but not in f and f_x. At the same time $|X| \sim |Z_f|$ should be satisfied. Combining this with the relation $|Y| \sim |Z_g|$ and Eq. (15), we find

$$\frac{n}{m} = \frac{k}{m-l} > 1 \tag{53}$$

and

$$x \sim \rho^{1/2(n-1)}, \tag{54}$$

$$y \sim x^{n/m} \sim \rho^{n/2m(n-1)}. \tag{55}$$

Finally, a large M-mass becomes

$$M_C|\omega_2| \sim m_{SUSY} \left(\frac{x}{y}\right)^2. \tag{56}$$

4 Minimization of Scalarpotential

Although in the previous section we find desirable solutions, a question arises as to whether or not the solution found there represents the absolute minimum of the scalarpotential. Then in this section we study the structure of the scalarpotential concretely and find an additional condition such that a desirable solution becomes the absolute minimum of the scalarpotential. Since we consider the case when the relation (53) is satisfied, we get $|X| \sim |Z_f|$ and $|Y| \sim |Z_g|$ coincidentally in the region $x^n \sim y^m$. Solving the stationary condition (24), one can find local minima and saddle

points of the scalarpotential. In this case, it can be proven for the scalarpotential with ${\rho_y}^2 > 0$ that there are the following two or three local minima. The values of the scalarpotential at these points are calculable.

Point A: $(x, y) = (x_0, y_0)$.

$$M_C{}^{-4} V \cong -\frac{4(n-1)}{(2n-1)}\, {\rho_x}^2 {x_0}^2, \tag{57}$$

where

$$\begin{aligned} x_0 &= \left(\frac{\rho_x}{\sqrt{2n-1}\, a\, \xi}\right)^{1/2(n-1)}, \\ y_0 &= b \left(\frac{c}{k}\right)^{1/2(m-l)} {x_0}^{k/(m-l)} \quad (\ll x_0), \\ \xi &= \left| 1 - \frac{k}{l} \left(\frac{c}{k}\right)^{n/k} \right| . \end{aligned} \tag{58}$$

Point B: $(x, y) = (x'_0, 0)$.

$$M_C{}^{-4} V \cong -\frac{4(n-1)}{(2n-1)} {\rho_x}^2 {x'_0}^2, \tag{59}$$

where

$$x'_0 = \left(\frac{\rho_x}{\sqrt{2n-1}\, a}\right)^{1/2(n-1)}. \tag{60}$$

Point C: $(x, y) = (x'_0, y'_0)$ (only for $l \geq 2$ and $1 + R \geq 0$).

$$M_C{}^{-4} V \cong -\frac{4(n-1)}{(2n-1)}\, {\rho_x}^2 {x'_0}^2, \tag{61}$$

where

$$\begin{aligned} y'_0 &= b \left(\frac{k^2 b^2}{(2l-1)c} \left(1 + \sqrt{1+R}\right)\right)^{1/2(l-1)} {x'_0}^{(n-k-1)/(l-1)} \quad (\ll y_0), \\ R &= -\frac{(2n-1)(2l-1){\rho_y}^2}{k^2 {\rho_x}^2} \quad (< 0). \end{aligned} \tag{62}$$

Point A is a solution which was obtained in the previous section and also found by Masip[8]. At this point not only two terms in $g(x, y)$ cancel out with each other in their leading order but also the leading term of $f\, f_y$ in Eq. (19) cancels out $g\, g_y$. In

the expansion the ratio of the next-to-leading terms to the leading ones is $O((y_0/x_0)^2)$. In the case $l \geq 2$ and $1 + R \geq 0$, Point C becomes a local minimum but not in the other cases. Although x and y are non-zero at Point C, Point C is not a desirable solution because M-masses are $O(m_{SUSY})$. In addition to local minima, we also have saddle points which are located at the origin and the following points.

Point D: $(x, y) = (x_1, y_1)$.

Where

$$
\begin{aligned}
x_1 &= \eta^{n(2m-1)/2k\phi} \left(\frac{(2l-1)b\rho_x}{2a\sqrt{k(m-l)(2m-1)}} \right)^{m/\phi} \qquad (\ll x_0), \\
y_1 &= b \left(\frac{(2l-1)b\rho_x}{2a\sqrt{k(m-l)(2m-1)}} \right)^{m/\phi} \qquad (\ll y_0)
\end{aligned}
\tag{63}
$$

with

$$
\begin{aligned}
\eta &= \frac{k(2m-1)}{(2l-1)c}, \\
\phi &= 2mn - m - n.
\end{aligned}
\tag{64}
$$

Point E: $(x, y) = (x'_0, y'_1)$ (only for $l \geq 2$ and $1 + R \geq 0$).

Where

$$
y'_1 = b \left(\frac{k^2 b^2}{(2l-1)c} \left(1 - \sqrt{1+R}\right) \right)^{1/2(l-1)} {x'_0}^{(n-k-1)/(l-1)} \qquad (\ll y_0). \tag{65}
$$

If ${\rho_y}^2 < 0$, we have two local minima at Points A and B for $l = 1$ and at Points A and C for $l \geq 2$.

In comparison of Eq. (57) with Eqs. (59) and (61), Point A becomes the absolute minimum under the condition $0 < \xi < 1$. This condition on ξ is translated as

$$
0 < c < k \left(\frac{2l}{k} \right)^{k/n} \quad \text{and} \quad c \neq k \left(\frac{l}{k} \right)^{k/n}. \tag{66}
$$

It is worth noting that under this condition the Point A is the absolute minimum independent of the sign of ${m_N}^2$. For illustration we show the behavior of the scalarpotential for the cases $(n, k, m, l) = (6, 3, 2, 1)$ and $(9, 3, 3, 2)$ in Figs. 1 and 2, respectively.

In these Figures the vertical axis is taken as

$$v = \left(2{M_C}^4{\rho_x}^2{x_0}^2\right)^{-1} V + 1 \tag{67}$$

and instead of x and y the horizontal axes are taken as $\overline{x} = (x/x_0)^{n/m}$ and $\overline{y} = y/y_0$ so that the point $(\overline{x}, \overline{y}) = (1,1)$ becomes the absolute minimum (Point A). In the case $(n,k,m,l) = (6,3,2,1)$ the condition (66) leads to $0 < c < \sqrt{6}$ and $c \neq \sqrt{3}$. Here we put $a = b = c = 1$ in Fig. 1 and $a = b = 1,\ c = 2$ in Fig. 2. As seen in Fig. 1, local minima (Points A and B) are located at bottoms of very deep canyons. This comes from the fact that a curveture along the direction perpendicular to the line $x^n = y^m$ represents a large M-mass squared. In the case $m = 2$ and $l = 1$ the canyon is most steep. In the other cases the slope of the canyon becomes gentle relative to the case $m = 2$ and $l = 1$. These situations are seen in Figs. 1 and 2.

Fig. 1

Fig. 2

We are now in a position to evaluate the M-mass matrix for $(S + \overline{S})/\sqrt{2}$ and $(N + \overline{N})/\sqrt{2}$ at the absolute minimum (Point A). The mass matrix is of the form

$$M_C\, A = M_C \begin{pmatrix} f_x & g_x \\ f_y & g_y \end{pmatrix} = m_{SUSY} \begin{pmatrix} O(1) & O(x_0/y_0) \\ O(x_0/y_0) & O((x_0/y_0)^2) \end{pmatrix}. \tag{68}$$

More precisely, the matrix elements are

$$\begin{aligned} f_x &= \rho_x \frac{1}{\sqrt{2n-1}\,\xi} \left[(2n-1) - (2k-1)\frac{k}{l}\left(\frac{c}{k}\right)^{n/k} \right], \\ f_y = g_x &= \rho_x \left(\frac{x_0}{y_0}\right) \frac{2k}{\sqrt{2n-1}\,\xi} \left(\frac{c}{k}\right)^{n/k}, \\ g_y &= \rho_x \left(\frac{x_0}{y_0}\right)^2 \frac{2(m-l)}{\sqrt{2n-1}\,\xi} \left(\frac{c}{k}\right)^{n/k}. \end{aligned} \tag{69}$$

Thus we obtain a large M-mass

$$M_{N'} = M_C\,\omega_2 = \frac{2(m-l)}{\sqrt{2n-1}\,\xi}\,(c/k)^{n/k}\,\sqrt{-m_S{}^2}\,(x_0/y_0)^2\,, \tag{70}$$

which is associated with the eigenstate

$$N' = \cos\theta\frac{1}{\sqrt{2}}(N+\overline{N}) + \sin\theta\frac{1}{\sqrt{2}}(S+\overline{S}) \tag{71}$$

with

$$\theta = -\frac{k}{(m-l)}\left(\frac{y_0}{x_0}\right). \tag{72}$$

The eigenstate with mass $M_C|\omega_1| = O(m_{SUSY})$ is given by

$$S' = -\sin\theta\frac{1}{\sqrt{2}}(N+\overline{N}) + \cos\theta\frac{1}{\sqrt{2}}(S+\overline{S}). \tag{73}$$

The enhancement factor $(x_0/y_0)^2$ in Eq. (70) depends on n and m as

$$(x_0/y_0)^2 \sim (1/\rho_x)^{(n-m)/(n-1)m} \tag{74}$$

with $\rho_x{}^{-1} = M_C/\sqrt{-m_S{}^2} = 10^{15\sim16}$. Since the exponent $(n-m)/(n-1)m$ decreases with increasing m, we take $m=2$ so as to get a sufficiently large M-mass $M_{N'}$. Then we have $l=1$ and $n=2k$. This leads to

$$M_{N'} = x_0\,O\left(\sqrt{M_C\times m_{SUSY}}\right). \tag{75}$$

Numerically we obtain

$$(x_0/y_0)^2 = 10^{7\sim8} \qquad \text{for} \;\; n\geq 6 \tag{76}$$

and the M-mass becomes

$$M_{N'} = O\left(10^{9\sim10}\text{GeV}\right) \tag{77}$$

by taking $\sqrt{-m_S{}^2} = O(10^3\text{GeV})$. Consequently, a large M-mass can be induced from the NR interactions of S, N and $\overline{S}, \overline{N}$ which are of the form

$$W_{NR} = M_C{}^3\lambda_1\left[\left(\frac{S\overline{S}}{M_C{}^2}\right)^n + \frac{n}{2}\left(\frac{N\overline{N}}{b^2\,M_C{}^2}\right)^2 - 2c\left(\frac{S\overline{S}}{M_C{}^2}\right)^{n/2}\left(\frac{N\overline{N}}{b^2\,M_C{}^2}\right)\right] \tag{78}$$

with $0 < c < \sqrt{n}$ and $c \neq \sqrt{n/2}$. For comparison we tabulate the orders of $\langle S \rangle, \langle N \rangle$ and $M_{N'}$ for several cases of the set (n, k, m, l) in Table I. As seen in this Table, unless $m = 2$ and $l = 1$, $M_{N'}$ attains to only at most $O(10^7\mathrm{GeV})$. The case $m = 2$ and $l = 1$, which leads to $n = 2k$, is indispensable for solving the solar neutrino problem.

Table I

5 Small See-saw Neutrino Masses

In the previous sections, we have constructed a consistent model with two large intermediate energy scales of symmetry breaking. The higher energy scale is given by the VEV $\langle S \rangle = \langle \overline{S} \rangle = O(10^{16\sim18}\mathrm{GeV})$ which can prohibit fast proton decay. The other energy scale is the VEV $\langle N \rangle = \langle \overline{N} \rangle = O(10^{13\sim15}\mathrm{GeV})$. These scales induce the large Majorana neutrino mass $M_{N'}$ with $O(10^{9\sim10}\mathrm{GeV})$.

In this section we propose a viable model which explains the smallness of three kind of neutrinos $\nu_e, \nu_\mu$and ν_τ. This problem could be reduced to see-saw mechanism [3]. The present experimental limits on neutrino masses are given as [14]

$$m_{\nu_e} < 7.3\mathrm{eV}, \qquad m_{\nu_\mu} < 270\mathrm{keV}, \qquad m_{\nu_\tau} < 35\mathrm{MeV} \tag{79}$$

by the laboratory experiments. On the other hand, recent experiments on solar neutrino and atomospheric neutrino have given more stringent constraints on neutrino masses and mixing parameters. From solar neutrino experiments by Homestake, Kamiokande and recent GALLEX [5] [15] [16] the allowed nonadiabatic narrow MSW band [4] is

$$\Delta m_{12}^2 \simeq (2.7 \sim 13) \times 10^{-6}\mathrm{eV}^2 \ , \qquad \sin^2 2\,\theta_{12} \simeq 0.004 \sim 0.013 \tag{80}$$

for the mixing among the first and the second generations. Atomospheric neutrino experiments by Kamiokande and IMB Collaboration[15][17] have shown the depletion

of the atmospheric muon-neutrino flux. The allowed neutrino oscillation parameters have been given in Ref.[15]. From this it is expected that the heaviest neutrino mass among three light neutrinos is $O(10^{-1}\mathrm{eV})$ and that there is a large mixing of the muon-neutrino with another neutrino. If we combine the solar neutrino data with the atomospheric neutrino ones, the possible mixing solution is given by

$$\Delta m_{23}^2 \simeq (2 \sim 40) \times 10^{-3}\mathrm{eV}^2 \ , \qquad \sin^2 2\,\theta_{23} \simeq 0.4 \sim 0.7 \ . \tag{81}$$

From these results the neutrino masses are

$$m_{\nu_\mu} \simeq (1.6 \sim 3.6) \times 10^{-3}\mathrm{eV}, \tag{82}$$

$$m_{\nu_\tau} \simeq (0.4 \sim 2) \times 10^{-1}\mathrm{eV}, \tag{83}$$

provided that $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$.

Here we are going to estimate the magnitude of large M-masses which lead to sufficiently small neutrino masses by see-saw mechanism. To do this, we need to know the Dirac-mass matrix for neutrinos. We take two possibilities for the structure of neutrino Dirac-mass matrix.

One possibility is that the leptonic Dirac-mass matrix is the same as the quark one from the standpoint of the quark-lepton unification at the Planck scale. We take masses of up-quark sector as Dirac-mass terms of neutrinos. From the above constraints on neutrino masses we obtain right-handed M-masses

$$M_{M_2} \simeq \frac{m_c^2}{m_{\nu_\mu}} \sim (0.6 \sim 1.4) \times 10^{12}\mathrm{GeV}, \tag{84}$$

$$M_{M_3} \simeq \frac{m_t^2}{m_{\nu_\tau}} \sim (1 \sim 6) \times 10^{14}\mathrm{GeV} \tag{85}$$

by taking $m_c = 1.4\mathrm{GeV}$ and $m_t = 150\mathrm{GeV}$. In this case we are compelled to get mass hierarchy also for right-handed M-mass matrix. To obtain a reasonably light neutrinos by using see-saw mechanism, we need at least two M-masses of $O(10^{12}\mathrm{GeV})$ and $O(10^{14}\mathrm{GeV})$ as derived above. In the present model, it is difficult to obtain a M-mass as large as $O(10^{14}\mathrm{GeV})$.

The other possiblity is that the structure of Dirac neutrino masses is the same as the one of charged lepton masses. In this case right-handed M-masses become

$$
\begin{aligned}
M_{M_2} &\simeq \frac{m_\mu^2}{m_{\nu_\mu}} \sim (0.3 \sim 0.7) \times 10^{10}\mathrm{GeV}, && (86)\\
M_{M_3} &\simeq \frac{m_\tau^2}{m_{\nu_\tau}} \sim (1.6 \sim 8.0) \times 10^{10}\mathrm{GeV}. && (87)
\end{aligned}
$$

Then a single M-mass scale with $O(10^{10}\mathrm{GeV})$ can reproduce the small neutrino masses consistent with recent solar and atomospheric neutrino experiments. Unfortunately, at present there is no theoretical basis that guarantees the equality $m_{\nu_i}(Dirac\ mass) \simeq m_{e_i}$, where m_{e_i} means i-th charged lepton mass. We now have no knowledge about Yukawa couplings $N_i L_j H_u$ at the compactification scale. So this similarity between neutral and charged Dirac-mass terms is an important subject that we should derive from superstring theory in the future study.

Now we propose a simple model with three generations along the scenario given in the previous section. First we consider a case in which all generations of right-handed sneutrino N_i develop almost the same VEV in magnitude, i.e.,

$$
\langle N_i \rangle = O(10^{13\sim 15}\mathrm{GeV}). \tag{88}
$$

This scenario is implemented by substituting $\Sigma_{i=1}^3 N_i$ for N in Eq. (78). However, the superpotential contains the Yukawa interaction term like $N_i L_j H_\mathrm{u}$, where the indices i, j mean the generation degree of freedom and we assume only one generation for Higgs sector below the scale $\langle S \rangle$. This term generates the large mixing masses for $L_j H_\mathrm{u}$ due to the VEV $\langle N_i \rangle = O(10^{13\sim 15}\mathrm{GeV})$. So these large mixing masses bring about the large Dirac-masses for neutrino states and then this model is inconsistent with the small neutrino masses.

Therefore, as an alternative to the above case, we next consider the case that the VEVs become

$$
\begin{aligned}
&\langle N \rangle = \langle \overline{N} \rangle = O(10^{13\sim 15}\mathrm{GeV}), \\
&\langle N_1 \rangle = \langle N_2 \rangle = \langle N_3 \rangle = 0. && (89)
\end{aligned}
$$

To construct a viable model it is assumed that we have the Yukawa interactions $N_i L_j H_{\rm u}$ but not $N L_j H_{\rm u}$. The NR interactions

$$W_{NR} = W_{NR}^{(0)} + W_{NR}^{(1)} . \tag{90}$$

implements this situation (89), where

$$W_{NR}^{(0)} = {M_C}^3 \lambda_1 \left[\left(\frac{S\overline{S}}{{M_C}^2} \right)^n + \frac{n}{2} \left(\frac{N\overline{N}}{b^2\,{M_C}^2} \right)^2 - 2c \left(\frac{S\overline{S}}{{M_C}^2} \right)^{n/2} \left(\frac{N\overline{N}}{b^2\,{M_C}^2} \right) \right] \tag{91}$$

with $0 < c < \sqrt{n}$ and $c \neq \sqrt{n/2}$ and

$$W_{NR}^{(1)} = \lambda_4 \left(\frac{N_1\overline{N}}{M_C^2} \right) \left(\frac{N_2\overline{N}}{M_C^2} \right) + \lambda_5 \left(\frac{N_3\overline{N}}{M_C^2} \right)^2 . \tag{92}$$

The superfields $N_i (i = 1, 2, 3)$ are contained in $W_{NR}^{(1)}$. The addition of $W_{NR}^{(1)}$ to $W_{NR}^{(0)}$ does not change the absolute minimum with the VEVs $\langle N \rangle = \langle \overline{N} \rangle = M_C y_0$ and $\langle S \rangle = \langle \overline{S} \rangle = M_C x_0$. Here it is assumed that there is no term like

$$\left(\frac{S\overline{S}}{M_C^2} \right)^{n/2} \left(\frac{N_i\overline{N}}{M_C^2} \right) \qquad (i = 1, 2, 3). \tag{93}$$

If W_{NR} contains this type of the NR terms, the VEVs at the absolute minima could change. Absence of these terms can be guaranteed by the introduction of discrete symmetries. For illustration let us take here $n = 6$. If the model contains the discrete symmetry $\mathbf{Z}_7 \times \mathbf{Z}_2$ and if each superfield has a suitable discrete charge as shown in Table II, the superpotential (90) to (92) is allowed whereas the terms (93) are forbidden. In Table II the discrete charge of Grassmann number is taken as $(-1,\ -)$. It is interesting for us to remember Gepner model in which Calabi-Yau manifold is constructed algebraically by a tensor product of $N = 2$ minimal superconformal models with the level k's [18]. In Gepner model there appears the discrete symmetry $\mathbf{Z}_{k+2} \times \mathbf{Z}_2 (\mathbf{Z}_{k+2})$ for each $N = 2$ minimal superconformal model with an odd(even) level k. In view of the fact that algebraic construction of compactified manifolds brings about various types of the discrete symmetry, the present model is a likely scenario.

Table II

From Eq. (92), we finally obtain the mass matrix for the Majorana neutrino sector as

$$
M_M = \begin{array}{c} \begin{array}{ccccc} N_1 & N_2 & N_3 & N' & S' \end{array} \\ \begin{pmatrix} 0 & \sim \lambda_4 M_{N'} & 0 & 0 & 0 \\ \sim \lambda_4 M_{N'} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sim \lambda_5 M_{N'} & 0 & 0 \\ 0 & 0 & 0 & M_{N'} & 0 \\ 0 & 0 & 0 & 0 & \sim m_{SUSY} \end{pmatrix} \end{array} . \tag{94}
$$

So all Majorana neutrinos have masses of order $M_{N'}$ except for the field S' which has the mass of $O(m_{SUSY}) \simeq O(1\text{TeV})$. Dirac-mass terms come from usual Yukawa interactions

$$
\lambda_{ij} N_i L_j H_{\rm u} \simeq \lambda'_{ij} E_i L_j H_{\rm d} \,, \tag{95}
$$

where E_i means i-th $SU(2)_L$-singlet charged lepton fields. Since $\langle H_{\rm u} \rangle \simeq \langle H_{\rm d} \rangle \simeq O(10^2\text{GeV})$, we obtain almost the same Dirac mass matrix for neutrinos as for charged leptons. So large M-masses induced by the above mechanism yield very small neutrino masses via the see-saw mechanism. The results are consistent with recent solar and atmospheric neutrino experiments.

6 Summary and Discussion

In Calabi-Yau superstring models with abelian flux breaking the gauge group is rank-six at the compactification scale. To connect Calabi-Yau models with the standard model, there should exist two intermediate energy scales of symmetry breaking between the compactification scale and the electroweak scale. In this paper, we clarified that in a certain type of Calabi-Yau superstring models the symmetry breaking occurs

by stages at two large intermediate energy scales which are given by $\langle S\rangle(\langle\overline{S}\rangle)$ and $\langle N\rangle(\langle\overline{N}\rangle)$. Two large intermediate scales induce a large M-masss of right-handed neutrinos. Peculiar structure of the effective NR interactions is crucial in models. Furthermore, the special sets $m=2, l=1, n=2k\geq 6$ for the NR interactions are necessary for viable scenarios, in which the NR terms of the superpotential become Eq. (78). In fact, the M-mass becomes $O(10^{9\sim 10}\text{GeV})$ for these cases and see-saw mechanism can be at work. We proposed a concrete model with three generations which leads to small see-saw M-masses for neutrinos. This large M-mass solves the solar neutrino promlem and also is compatible with the cosmological bound for stable light neutrinos. Special form of the NR terms suggests that the superstring model possesses an appropriate discrete symmetry coming from distinctive structure of the compactified manifold.

Mass hierarchy of quarks and leptons may also have its origin in the discrete symmetry and the presence of large intermediate scales. Provided that mirror superfields except for S and N are not contained in the model, we may have distinct types of the NR terms, for instance, associated with the up-quark sector as

$$\sum_p \lambda_{ij}^{(p)} \left(\frac{S\overline{S}}{{M_C}^2}\right)^p Q_i U_j^c H_u\,, \tag{96}$$

where $\lambda_{ij}^{(p)}=O(1)$ and Q_i and U_j^c stand for quark-doublet and up-quark-singlet superfields for the i-th generation, respectively. If the discrete symmetry compel us to retain only the terms

$$\lambda_{11}^{(2)} \left(\frac{S\overline{S}}{{M_C}^2}\right)^2 Q_1 U_1^c H_u + \lambda_{22}^{(1)} \left(\frac{S\overline{S}}{{M_C}^2}\right) Q_2 U_2^c H_u + \lambda_{33}^{(0)} Q_3 U_3^c H_u \tag{97}$$

for the up-quark sector of the superpotential, we have the mass hierarchy of up-quarks such as

$$m_u \sim \langle H_u\rangle {x_0}^4, \qquad m_c \sim \langle H_u\rangle {x_0}^2, \qquad m_t \sim \langle H_u\rangle. \tag{98}$$

Since ${x_0}^2 \sim 10^{-(2.0\sim 2.3)}$ for $n=8$, we obtain a plausible solution which is in accord with experimental data. At all events the effective NR interactions play an important role in connecting the superstring theory with the standard model. It is the discrete

symmetry of the compactified manifold that controls the characteristic features of the NR terms.

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Table Captions

Table I

The energy scales of symmetry breaking $\langle S \rangle$ and $\langle N \rangle$ and a large Majorana-mass $M_{N'}$ in GeV unit for various cases of (n, k, m, l). Here we take $M_C = 10^{18.5}$GeV and $\sqrt{-m_S{}^2} = 10^3$GeV.

Table II

The charge assignment of the discrete symmetry $\mathbf{Z}_7 \times \mathbf{Z}_2$ for superfields $S,\ \overline{S},\ N,\ \overline{N}$, and N_i. The discrete charge of Grassmann number is taken as $(-1,\ -)$. $\mathbf{Z}_7 \times \mathbf{Z}_2$ is taken as only an example of the discrete group.

Figure Captions

Fig. 1

The structure of the scalarpotential in the case $(n,k,m,l) = (6,3,2,1)$ with $a = b = c = 1$. The vertical axis is taken as the normalized scalarpotential v (see text). The horizontal axes are $\overline{x} = (x/x_0)^3$ and $\overline{y} = y/y_0$, where $x = \langle S \rangle / M_C$ and $y = \langle N \rangle / M_C$.
(a) The overview of the scalarpotential v. The Point A (the absolute minimum) is located at $(\overline{x}, \overline{y}) = (1,1)$ and the Point B is a local minimum.
(b) The comparison of values of the scalarpotential v between Point A and Point B. A solid (dashed) curve represents the calculation of v vs. $\overline{x}$ along the line $\overline{x} = \overline{y}$ ($\overline{y} = 0$).
(c) The comparison of v vs. $\overline{y}$ along the line with fixed $\overline{x}$-values.

Fig. 2

The structure of the scalarpotential in the case $(n,k,m,l) = (9,3,3,2)$ with $a = b = 1$ and $c = 2$. The vertical and horizontal axes are taken as the same as in Fig.1.
(a) The overview of the scalarpotential v. The Point A (the absolute minimum) is located at $(\overline{x}, \overline{y}) = (1,1)$ and the Point B is a local minimum.
(b) The comparison of values of the scalarpotential v between Point A and Point B. A solid (dashed) curve represents the calculation of v vs. $\overline{x}$ along the line $\overline{x} = \overline{y}$ ($\overline{y} = 0$).
(c) The comparison of v vs. $\overline{y}$ along the line with fixed $\overline{x}$-values.

Table I

n	k	m	l	$\langle S \rangle$ (GeV)	$\langle N \rangle$ (GeV)	$M_{N'}$ (GeV)
4	2	2	1	$10^{15.9}$	$10^{13.1}$	$10^{8.1}$
6	3	2	1	$10^{16.9}$	$10^{13.5}$	$10^{8.8}$
8	4	2	1	$10^{17.4}$	$10^{13.6}$	$10^{9.1}$
10	5	2	1	$10^{17.6}$	$10^{13.7}$	$10^{9.2}$
12	6	2	1	$10^{17.8}$	$10^{13.7}$	$10^{9.2}$
20	10	2	1	$10^{18.1}$	$10^{13.7}$	$10^{9.2}$
6	4	3	1	$10^{16.7}$	$10^{14.7}$	$10^{6.6}$
9	6	3	1	$10^{17.5}$	$10^{15.3}$	$10^{6.4}$
12	8	3	1	$10^{17.8}$	$10^{15.4}$	$10^{6.6}$
6	2	3	2	$10^{16.9}$	$10^{15.2}$	$10^{5.4}$
9	3	3	2	$10^{17.5}$	$10^{15.2}$	$10^{5.8}$
12	4	3	2	$10^{17.8}$	$10^{15.3}$	$10^{5.9}$

Table II

Fields	$\mathbf{Z}_7$-charges	$\mathbf{Z}_2$-charges
S	1	$+$
$\overline{S}$	1	$+$
N	3	$+$
$\overline{N}$	3	$+$
N_1	2	$-$
N_2	4	$-$
N_3	3	$-$
(θ)	-1	$-$